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Nine propositions about future-oriented mathematics education (FOME): An Introduction

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In the years before and after the turn of this new century or millennium, the mathematics education community was, and continues to be, adrift in books and conferences about the future. For example, the theme of the 8th Southeast Asian Conference on Mathematics Education held in Manila in 1999 was "Mathematics for the 21st century." In the year 2000, Universiti Brunei Darussalam held its fifth annual international conference with the theme "Science, Mathematics and Technical Education in the 20th and 21st century." In the same year, the National Council of Teachers of Mathematics (NCTM) in the United States published its yearbook with the title "Learning Mathematics for a New Century." I shall refer to these activities as evidence of the phenomenon called "future-oriented mathematics education" (FOME). This special issue of *The Mathematics Educator* is another example of FOME.

In this paper, I propose nine propositions about FOME, touching on its nature, teaching, and research, with some reference to my experience of mathematics education in Southeast Asia and Australia. I will limit myself to FOME up to the year 2020. This particular year coincides with Malaysia's *Vision 2020* and is only a few months after *July 20, 2019*, a futuristic book by Arthur Clarke (1986). Beyond the year 2020, there are simply too many possibilities that the visions become rather fuzzy. I offer only titbits rather than a comprehensive review of this vast and challenging area.

Proposition 1: FOME provides more visions than predictions

There are two aspects to FOME: *predicting* what school mathematics is likely to be in the future and expounding *visions* of what it ought to be. Predicting is much harder to do and has a higher probability of failure than expounding a *vision*. Many predictions about even short-term changes often turn out to be wrong. For example, in the past it was (and still is) believed that doing mathematics does not incur much expenditure, requiring only paper and pencil. This has changed in the past two decades as much serious mathematics is now done using expensive computers, the notable examples being the proof of the Four Colour Theorem in 1976 and the study of chaos and fractals. On the other hand, to provide a *vision* is relatively straightforward because it does not require serious considerations of the practical situations and the unexpected political, social, and economic changes that will occur in the future. This explains why mathematics educators are offering more visions than predictions.

One might predict that by the year 2020, many schools in this region will still not be equipped with good facilities for teaching and learning; the mathematics contents and teaching methods will be very similar to the current situations; competent mathematics teachers will still be in short supply; and the Local Examinations Syndicate of Cambridge University will pass some of the profit it makes administering external examinations back to the host countries. As I write down these predictions, I am fully aware of the caution mentioned by Carl Sagan (1997), the eminent astronomer: "the prediction I can make with the highest confidence is that the most amazing discoveries will be the ones we are not today wise enough to foresee" (p. 52). Even in mathematics which has a strong claim of certainty, there are many results which were once proclaimed by renowned mathematicians of the past to be impossible "have been converted into active, useful, and logical possibilities" (Davis, 1987, p. 171). What more with FOME which deals with human behaviours!

Proposition 2: The FOME syndrome is contextualised by expertise and system

The FOME syndrome is more prevalent among mathematics educators (I will use this term to refer to tertiary lecturers) than school teachers, and among Western educators more than Eastern educators. What is proposed in FOME may not transfer readily from the ivory towers to the dusty classrooms or from the West to the East (or vice versa).

To engage in FOME, one must be knowledgeable about mathematics education trends and issues that spread across decades if not centuries. In most universities, mathematics educators teach student teachers and practising teachers about these trends. With an understanding of these trends, teachers are supposed to be better prepared to face the future. Unfortunately, this rarely happens because most teachers are more concerned about tackling pressing day-to-day problems in the present than thinking about future visions.

Mathematics educators are more competent than teachers in discussing FOME because they have been trained as researchers to analyse, criticise, and look for alternative viewpoints. They often conclude their research reports with a section on "Implications for practice and future research". Compared to school teachers and ministry officials, mathematics educators also have more time to reflect, wider exposure through networking and attending conferences, and greater access to journals and books. Thus, they have ample practice in FOME.

Much of FOME is written by Western scholars. Asian mathematics educators have not been very active in this endeavour. One exception I would like to mention is the keynote address by Lee Peng Yee at Manila in 1999. He would like to see in the future, less content for more students, a combination of manipulative skills and problem solving skills, new learning strategies in the IT culture, and students doing mathematics experimentally. The difference in output about FOME from the West and the East can be explained by the respective education environments. America and Australia have no national curriculum, so there is a wider scope for mathematics educators in those countries to offer their insights about the future. In contrast, practically all the Asian education systems are centralised, and some, like Singapore and Brunei Darussalam, use an external mathematics curriculum. Hence, it is pointless for the majority of educators and teachers in these countries to engage in FOME. The energy could be better spent in developing and adapting innovative teaching materials and approaches to solve local problems. This is a case of "global connections and local solutions".

FOME cannot be divorced from the general education system of which it is an integral part. It is constrained by the tradition, structure, philosophy, values, and practical matters that each education system has to deal with. For example, the call to include more statistics in the mathematics curriculum is probably an American concern (Scheaffer, 2000). In contrast, there is already adequate amount of statistics in the primary and secondary mathematics curriculum in Brunei, Malaysia, and Singapore. Minor topics such as stem-and-leaf or box plot may be included. On the other hand, what mathematics curriculum and pedagogy are suitable for the Special, Express, or Normal streams is a unique Singapore concern that is best worked out by local educators. However, there are some issues that affect many countries and these will benefit from joint international effort. For example, it has been found in Australia (Bagnall, 2001) and Brunei that girls are performing *better* than boys in many subjects. In this instance, FOME will have to address the reversed gender issue (improving boys' performance in mathematics) in the coming years.

Proposition 3: Engaging in FOME satisfies a need of wanting to make a contribution to the future

Knowledgeable and experienced mathematics educators are well placed to steer mathematics education for the future generations. They want the next generations to be better in mathematics. Thus, to engage in FOME satisfies some inherent psychological need similar to that of prophets in religions. Similar to religious teachings, some FOME mission statements reflect this sentiment. For example, the politicians and educators in the United States want their students to be the first in the world in mathematics and science. Such a statement has a visionary quality and may be difficult, if not impossible, to achieve, but this does not deter many from proposing reforms with that goal in mind.

Proposition 4: FOME should provide micro-analysis of the present to the future

FOME writings contain many general statements, but the details are yet to be worked out, presumably by the teachers and curriculum planners. Most people will agree to sensible general principles but may disagree, sometimes quite vehemently, about the details of how these principles are applied at the systemic, school, and classroom levels.

Let me illustrate this by considering the teaching of subtraction with renaming, say 62 – 28. To arrive at a full understanding, the pupil needs to know at least the following.

Concepts	(C1) The meanings of the symbol and operation.
	(C2) Illustrate the subtraction using hands-on activities.
Algorithms	(A1) Apply the "borrow" or "rename" procedure correctly.
	(A2) Illustrate the steps of algorithm (A1) using manipulatives. Here I am thinking of activities that involve exchange and not counting 62 marbles and removing 28 marbles, because this latter procedure does *not* mimic the algorithm.
	(A3) Explore alternative algorithms, if available.
Check answer	(Ch1) Check the answer using estimation, say 60 – 30.
	(Ch2) Check the answer using a calculator.

Applications	(Ap1) Solve word problems.
	(Ap2) Find and solve problems in everyday life and other subjects that require subtraction.
	(Ap3) Pose own problems about subtraction.
Values	(V1) Make decisions about subtractions in a rational way.
	(V2) Use subtraction to deal with real life problems in an ethical way.

In current classroom teaching, many primary school teachers will do C1, briefly demonstrate C2 or skip it, spend a lot of time on A1, skip A2, A3, and Ch1, do Ch2 if calculator is allowed, do Ap1 but not as much as A1, skip Ap2 and Ap3. The values objectives hardly enter into the consciousness of most mathematics teachers, even though some mathematics syllabuses (e.g., Malaysia and Brunei) mention the inculcation of desirable values. The outcomes are well known: many primary school pupils are confused by the algorithm (A1), become bored with doing numerous drills, and cannot solve word problems (Ap1), confounded by weakness in language. Research like Ma's (1999) study shows that American primary school teachers did not understand the rationale of the algorithm (A2) because they themselves were not taught C2. Thus, algorithm is a huge stumbling block to many pupils and teachers. I also believe that a mastery of algorithms does not necessarily advance mathematical thinking and usage. Indeed, in a recent study by Pesek and Kirshner (2000), for fifth-grade pupils, instrumental learning of perimeter of quadrilaterals by formula *prior* to relational learning resulted in poorer test results when compared to relational learning only. This might have happened to teachers who learned mathematics instrumentally.

What does FOME offer in terms of algorithms versus concepts? It is widely held that calculator activities can promote A2, with the assumption that A1 should be limited to only small numbers but not abandoned entirely. This is a weak position. A strong one is to stop teaching the subtraction algorithm to 90% of the pupils. They can use calculators (or the latest gadget) to do the subtraction so that they can concentrate on concepts, check answer, and applications. This will promote mathematical thinking and enhance pupils' motivation. The remaining 10% who are mathematically more able may work with the algorithms to develop procedural thinking. *All* students, however, should develop values such as being rational (rather than acting on impulse), helping others, team spirit, open-mindedness, and honesty, through their mathematical lessons (see other examples in Bishop, 2001; Taplin, 1998; and a critical analysis by Jurdak, 1999). This is rarely discussed in

FOME, probably because there are few exemplars of how to achieve these values objectives in mathematics lessons.

The above micro-analysis of a mathematical procedure and the proposal to drop algorithms for the majority of students can be applied to almost all algorithms: let the machine do the tedious algorithms and let the students work with the input and output. The technology is already here and will become even more powerful and more readily available by 2020. To make this proposal feasible, the instructional objectives must be clearly spelt out and assessment must deal with the other aspects mentioned earlier. I will briefly mention a case to show how such a proposal may be inappropriately applied. Dr *X* taught a class of pre-service Certificate in Education students in Brunei. One of the course objectives was to improve the algebraic manipulative skills of the students. However, Dr *X* taught the students to use *Mathematica* to do all these manipulations, thereby replacing algebraic skills with pressing the correct icons on the software. In the examination paper, all the skill questions could be solved by simply entering the expressions in *Mathematica*, without much thinking about concepts or applications. As a result, this course objective was hijacked by a convenient but mindless use of technology. One should be on guard against such a mismatch between curriculum objectives, teaching methods, and assessment. This mismatch is also one of the five challenges raised by the Research Advisory Committee of NCTM (2000) for the coming years.

Whether one agrees or disagrees with the micro-analysis of the subtraction algorithm mentioned above is not the main point. Rather, more importantly, it is this kind of micro-analysis that I feel should be the norm when one attempts to plan for the future under FOME.

With respect to algebraic skills, despite more than ten years of discussion about how computer algebra systems (CAS) might change the algebra curriculum in schools, this has not yet taken place in most countries. The issue of making changes within a stable education environment is explored under the next proposition.

Proposition 5: FOME is hindered by stable contents and practices

Primary school mathematics curriculum is virtually the same all over the world. This testifies to the typical mathematical development of the human brain across different races and cultures, as expounded by the Piagetian theory of cognitive development or the van Hiele's theory for geometric thinking. Thus, FOME will have little impact in primary mathematics beyond suggestions such as to reduce the amount and complexity of fraction computations. Of course, local beliefs will make some changes to the general pattern. For example, a strong bias towards basic numeric skills in Malaysia has led to the virtual elimination of geometry in its primary mathematics syllabus.

For secondary school mathematics, there are considerable variations across countries. Within each system, the stability of the mathematics curriculum is quite apparent. For example, Cooper (1994) commented that the list of topics provided in the Cockcroft Report in 1982 "is very similar to that set out for the senior elementary school in the 1930s" (p. 10), despite a lapse of fifty years in between. My involvement in the revision of the national mathematics curriculum in Malaysia (1979), Singapore (1989), and Brunei (1999) has convinced me that it is very difficult to include new mathematics contents, such as topology, Euler's formula, and box plots, into the revised curriculum. Despite the current intense interest in fractals, it is surprising that none of the three mathematics curricula mentioned above has included this topic in its latest revision. It seems that much of the new mathematics developed in the past fifty years (see Shirley, 2000, for a brief coverage) will not be introduced into school mathematics by the year 2020! Indeed, according to Alan Bishop (1999):

> mathematics curricula … offer the students very little of a vision for the future. … many of the programs on offer merely reproduce what was done in pre-technology days. There are some new and exciting virtual-reality based programs available, but they are few and far between, and are only rarely used in the normal mathematics classroom. (p. 37)

My proposal is that mathematics in secondary schools should be presented to students as a smorgasbord of mathematics topics rather than *a la carta* menu with limited or no options. For each topic on offer, links to prior and post topics and the level of difficulty are made explicit so that the students can choose the most optimal sequence to learn these topics according to their interests and ability. This will eliminate the frustration many students now feel when they are forced to learn mathematics in a fixed sequence. Surprisingly, most mathematics curriculum documents, including the tome, *Principles and Standards for School Mathematics,* by NCTM (2000), show only the sequence of topics within each major area such as Numbers, Algebra, Measurement, or Geometry, but no links are given for topics across these major areas. Thus, many teachers will teach *all* the number topics, then *all* the algebra topics, and so on, rather than mix the various topics in a more integrated or spiral sequence. This gap in explicating the hierarchical structure of *all* the topics in the mathematics curriculum is a much-neglected aspect of FOME.

The above suggestion of "on-demand mathematics" may lead to the abolition of mathematics as a *separate* subject in schools. This may set off an alarm among some mathematics educators and policy makers! Under the new proposal, instead of certifying that students have passed mathematics, it is more important that they can demonstrate using various mathematical skills how to solve problems. This has challenging implications for assessment and school organisation. As the Taiwanese technopreneur, Sayling Wen (1999), wrote, with a far-sighted vision:

> When computer technology is introduced into the educational system the present knowledge-oriented, progressive promotion system can be replaced with the balanced development of multidirectional education. We can move from high cost centralised same-pace learning to decentralised individualised learning. (p. 50)

This calls for a drastic re-organisation of the school systems in the coming decades, to which FOME has not paid sufficient attention.

Will these FOME suggestions be realised by 2020? After reviewing changes to education in the 20th century, Jeremy Kilpatrick and George Stanic (1995) concluded that, despite the ebbs and flows of changes in school mathematics, "it has yet to achieve a form substantially different from that being established in the closing years of the last century" (p. 14). They cited David Eugene Smith, considered the first mathematics educator, who wrote in 1922: "The number of times the teaching of mathematics has been reformed and the general similarity of versions of reforms are always interesting to the student of the history of the subject." A similar conclusion was stated differently in a recent International Commission on Mathematical Instruction (ICMI) study. Its editors, Anna Sierpinska and Jeremy Kilpatrick. (1998), wrote: "[R]eforming curriculum on fundamental concepts, producing new teaching materials, and designing teaching units based on 'mathematicians' common sense' (or on logic) have not resulted in a better understanding of mathematics by students" (p. ix). They argued that theories in mathematics education "must deal simultaneously with cognition, social institutions, and mathematics in their complex mutual relationships" (ibid.). This task is truly awesome.

While changing the contents of mathematics curriculum is difficult, it is even more difficult to change how mathematics is taught. Yet, FOME has offered many suggestions on how mathematics should be taught in the future: use cooperative learning, emphasise mathematics communications using journal writing and oral presentations, teach with constructivist or de-constructivist theories, do away with textbooks or co-author textbooks with students (Healy, 1993), let students decide what to learn, use authentic assessment, and so on. To illustrate, I will refer to an interesting book by Holton, Holton, and Pedersen (1997). They mentioned eight general principles and four specific principles on how to *do* mathematics and five principles on how to *teach* mathematics. General principle 1 states, "Mathematics is only done effectively if the experience is enjoyable" (p. 324); specific principle 2: "Be optimistic!" (p. 332); pedagogical principle 5: "Where there are at least two different ways of looking at a problem, discuss at least two" (p. 337). These are sound principles, but will they work in Asian classrooms with large class sizes?

Asian teachers are often told to curtail the use of traditional chalk-and-talk and rote learning (however, see Biggs & Watkins, 1995, for a discussion of rote learning in the Asian context; also Dimmock, 2000). Note that teaching practices are ingrained in out-of-school cultural mores and are thus resistant to changes. A very strong Asian perception about teachers is that they are the authority in the classroom and they should transmit knowledge, mathematics or otherwise, to the students (Wong, Zaitun, & Veloo, 2001). It is not easy for Asian teachers to relinquish this authority or for Asian students to accept alternative roles for their teachers, such as being an organiser or a facilitator. This is, indeed, a special FOME issue for Asians.

Even if teachers, as well as mathematics educators, agree with most of the "progressive" pedagogical principles mentioned earlier, their practices are often at odds with theories. Pagano (1991) explained the mismatch in this way:

> We know how often teachers' practices appear to be at odds with the theory they profess. … One reason is that we are not accustomed to theorizing daily life. Theory exists on some metalevel, in some domain where thought and logic rule. But daily life is often so noisy that we can't even hear ourselves think. The other reason is that the language of theory is rarely the language of things. (p. 194)

To move beyond the rhetoric to the specifics about new ways of teaching mathematics is a challenge. In this endeavour, many mathematics educators believe that information and communications technology (ICT) will be a powerful asset. This leads to the next proposition.

Proposition 6: Information and communications technology is seen as the crucial factor in FOME

The past few years have seen the emergence of just about everything attached with an e-prefix to give it a futuristic aura: e-commerce, e-government, e-technology, e-book, e-magazine, e-learning, and so on. According to a *Time* report (June 4, 2001), 3 to 5 year old children in Germany are attending classes on how to manoeuvre the mouse before they can write properly with pencil (will manual writing soon become an obsolete skill?). Barbie B-Book that looks like a laptop, can talk in Barbie voice, and is loaded with educational games, is widely marketed to encourage girls to learn about computers. In Singapore, pre-schoolers from 230 kindergartens learn spelling and mathematics using computers (*Straits Times, Computer Time* section, July 18, 2001). One negative impact of too early and too much exposure to computers is already evident: these toddlers want fast responses and few will have the patience for slow lessons. Will they persevere when solving difficult mathematics problems? A recent report from the Alliance for Childhood claims that exposing children to computers at an early age may hamper their intellectual growth and endanger their health (Cordes & Miller, 2000). These are some of the changes in student characteristics that FOME has to contend with. By 2010, these computer savvy toddlers will become adolescents and by 2020, some of them may become mathematics teachers. Will they benefit from the FOME visions that are being *discussed* now but will take many years to implement?

The large number of papers and books published about ICT and mathematics education highlights the underlying sentiment that ICT will achieve what cannot be done using more traditional methods. However, Bishop's comment (1999) cited earlier suggests that innovative ICT programs are still rare and have not yet become part of the mainstream instruction. The efficacy of these programs is still being studied.

Nevertheless, mathematics teachers are nowadays exhorted to use ICT in their lessons: spreadsheet to help students solve problems involving real data, dynamic geometry to promote geometrical thinking, java applets to work on virtual manipulatives, computer algebra system to reduce algebraic manipulations, multi-sensory tutorials with game-like features to motivate and entertain, computer-mediated testing, search the Internet for resources, use email for networking and submission of homework, and so on. With dramatic advances in artificial intelligence, it will soon be possible to develop smart textbooks that will automatically adjust the materials presented to make students feel motivated to learn because the materials satisfy their interests, are set at their ability level, and are presented at different paces. The enormous effort expended by NCTM to develop e-materials is a clear indication of this trend. FOME will no doubt continue down the ICT path.

There are, however, pitfalls that should be avoided down this path. First, the mismatch between curriculum objectives, teaching practices using technologies, and assessment has already been illustrated with the case of Dr *X* under Proposition 4. Next is the seduction of the bells-and-whistles of technologies as they often distract mathematics teachers and educators from the important goals. To my knowledge a polytechnic in Singapore requires most of the lectures to be delivered using PowerPoint, that ubiquitous tool used not only in business presentations but also in education conferences worldwide. Ideally, PowerPoint should be a powerful adjunct to a well-delivered oral explanation of a topic. In actual practice, many speakers just read the slides verbatim. Reading mathematics in this way will not help students who struggle to make sense of the symbolism and understand the thought processes when one tackles a difficult mathematics problem.

Another problem is the incredibly fast changes in technologies. In the next decade, "wearable" or wireless ICT devices will become common. In this scenario, mathematics-enabled tools will be available anywhere and anytime. The *applications* of these tools to solve real-life problems will be more important than the ability to carry out algorithms. Given this perspective, whether graphic calculators (cheaper, easy to carry around, but poor screen output) or flash calculator (see Waits & Demana, 2000) or computers (more expensive, fixed, but more versatile) should be used in mathematics lessons is a minor issue. More importantly, how do teachers use whatever technologies are available to achieve many of the worthwhile FOME goals mentioned under Proposition 4?

Many educators have already sounded certain caveats about the indiscriminate use of the Internet in education, including biased or inaccurate information, copying without understanding, plagiarism, and waste of time and effort to search through huge databases for some insignificant information. Teachers who assign students to do mathematics projects using Internet resources will have to deal with these issues creatively. Teacher education becomes an important factor to make technology-related FOME a reality.

Proposition 7: Prepare teachers adequately to handle FOME with creative teacher education programs

Bill Gates (1999) displayed some good insights about teachers when he wrote that teachers are "excited about anything that will help kids learn. What teachers don't want is to be thrown into something they have not had the opportunity to learn about and become comfortable with" (p. 388). Research has shown that most teachers, especially the veterans, require training to master ICT and to use it to teach more effectively. In many countries, such training is available to only a small number of dedicated teachers, while the majority of the teachers have not been trained. Even those who have been trained may not use ICT in their lessons citing obstacles such as time-table schedule, insufficient number of computers, lack of computer-based materials, and the hectic and exhausting classroom manoeuvres involved in moving from one computer to another fixing technical problems.

As mentioned under Proposition 5, teachers are often resistant to changes, not only in using ICT but also in other areas. They must be convinced that the proposed changes will improve students' learning or enhance their professional work and personal life. When this is not true, teachers will find ways and means to continue with the status quo. This stability phenomenon is also evident in mathematics teacher education programs. According Graham, Li, and Buck (2000), the current curriculum structures and requirements of some mathematics teacher education programs in the United States are traditional and are "similar to the ones that have been in place for at least the last 75 years" (p. 27). When changes are introduced in mathematics teacher education programs, do they always have the intended aims of preparing more competent mathematics teachers? Consider the following case about Dr *X*.

An important FOME message is that students should develop mathematical thinking. One aspect of this thinking is that mathematical definitions can be quite arbitrary. Dr *X* tried to teach a class of student teachers, whose mathematics ability, as measured by the O-level results, was just average, that one can define *parallel* lines in a number of different ways and then deduce some mathematics properties based on the definition chosen. This is indeed a laudable objective. The definition he chose to use was: two lines are *parallel* if they do not meet *within the boundary of the paper* containing the lines. He then led the student teachers to explore properties of corresponding angles, alternate angles, and so on, given this definition. What happened later on was that some of these student teachers taught their primary school pupils *this* unconventional definition instead of the traditional one. This approach, though with good intention, has led to some harmful outcomes because the trainer did not give due consideration to the ability of the trainees.

This case highlights the need to know more about how mathematics educators train pre-service and in-service teachers, beyond what is given in program documents, which are open to multiple interpretations and different implementations. For example, is writing essays a useful exercise to help student teachers become better mathematics teachers? Have mathematics educators succeeded in developing problem solving and mathematical thinking among trainees using cooperative learning techniques, constructivist approaches, ICT-related resources, or whatever innovative methods they preach to their trainees? There is hardly any research about the work of mathematics educators, in contrast to the large corpus of research conducted *on* mathematics teachers and students. FOME ought to take a closer look at this gap. Mathematics educators occupy the pinnacle of a hierarchy of power and authority in the field of mathematics education. They train the teachers, advise policy makers, and make decisions about mathematics education that have far reaching impacts. Yet, in most countries, their work is not supervised in a way similar to that of the teachers. As pointed out by Lagemann (2000), educational researchers have not developed "a strong, self-regulating professional community" (p. ix), which is also true of mathematics educators. By the year 2020, I do not believe this authority structure will change, unless the mathematics educators themselves, acting as professionals like doctors, lawyers, and engineers, take steps to be more accountable for what they do and how they do their work.

Proposition 8: Mathematics education research can illuminate FOME but has little impact on practice

Most of the writings on mathematics education research are done and read by other mathematics educators rather than teachers and policy makers who do not have ready access to these materials. Even training teachers to do research may not change their practices. One MEd student confided to me that he will continue to teach mathematics as before, even though he is now more knowledgeable about mathematics education research. Research findings do not translate readily into practices.

To probe more deeply into this unsatisfactory situation, one needs to consider the types of research conducted in mathematics education. Mathematics education research has changed from the scientific, quantitative types in the 1950s to the qualitative, interpretative ones in the 1980s. Given this trend, papers in mathematics education research change from tables and statistics to selective transcripts of interviews with students and teachers, glimpses of classroom dialogues, and thumb-nail copies of student work. Bias and prejudice in mathematics education research become evident when reviewers for journal submissions :

> dismiss certain statistical methods because at some point in the past these valuable research tools have been misused by well-meaning but incompetent researchers. Yet the dissatisfaction in the field over the years with the results of quantitative methods is certainly matched by the dissatisfaction with the results of some of the qualitative ones – and in any case, abuse of a research method is not a reason to abandon it. (Hanna, 1998, p. 406).

A creative mix of both methodologies seems to be the vision for FOMER (see Pesek and Kirshner, 2000, for a recent study of this mix). In terms of content, a survey by Lubienski and Bowen (2000) of 3011 articles in mathematics education research in the past 16 years found that 20% were on integers, 17% on problem solving, 15% on geometry, 15% on student characteristics, 15% on technology, 6% on teacher education, and 5% on student assessment. The predominance of research on mathematics content is likely to continue because these studies might provide some useful insights for teachers.

To enhance the applicability of research, it is necessary to further explore the relationships between development, research, and dissemination. These relationships are shown in the figure below.

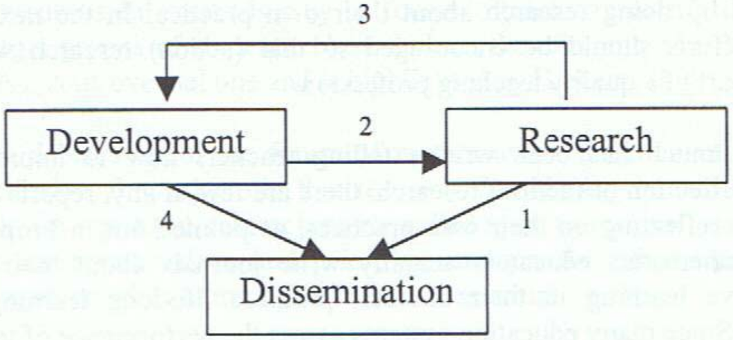

Figure1: Components of the educational research process

Much of the current quantitative and qualitative researches are descriptive and correlational in nature. These types of research make use of questionnaires and tests that are rarely used again after the research has been completed. This is shown by path (1) in the figure. The findings have certainly enriched our knowledge about some fairly focussed (or narrow) areas, but as mentioned by Pagano (1991), this knowledge can rarely be used to solve practical problems or determine policy matters.

Experimental studies often require the development of materials and innovative teaching techniques. This follows the path (2) → (1). To enhance the findings, path (3) should be included so that the materials and techniques are continually refined through replications in different contexts. Unfortunately, this is rarely done, thus weakening the claims of experimental studies to guide practices. Path (4) is the situation when new materials and techniques are disseminated without the benefits of research. This is especially true with textbook writers, who seem to exert a much stronger and more direct influence on FOME than the other paths.

In recent years, the teachers-as-researchers movement has gathered momentum in many countries. The intention is to encourage teachers to be reflective practitioners (Schön, 1983): do not just follow the textbooks or the curriculum, do not teach in the ways they have been taught before, experiment and be inquisitive about their practices, reflect on problems and solutions, and so on. In Brunei Darussalam, the first *Mathematics Teachers Conference* was held in 1999. This conference was organised by secondary school mathematics teachers and all the papers were written and presented by the teachers (Wong et al., 1999). This illustrates that when given proper guidance, mathematics teachers can reflect on their experiences and generate some theories in action. Through this process, teachers will become less mystified by doing research about their own practice. In the next two decades, similar efforts should be encouraged so that (action) research will become an integral part of a quality teaching profession.

Although much has been written telling teachers how to improve their work through reflection or (action) research, there are few, if any, reports of mathematics educators reflecting on their own practices, as pointed out in Proposition 7. How many mathematics educators actually write journals about their practices, use cooperative learning in their courses, promote life-long learning among their trainees? Since many education systems assess the performance of teachers through percentage passes of their students (no doubt a controversial and not a very popular practice), the same yardstick could apply to assess the performance of mathematics educators through the performance of the teachers they have trained. Research along this line is a worthwhile FOMER agenda for the next two decades, and the results might even enhance the professional standing of mathematics educators.

At the Ninth International Congress on Mathematics Education (ICME-9) held in Japan last year, Professor Mogens Niss, in his plenary lecture entitled "Key issues and trends in research on mathematical education" urged mathematics educators to find ways to reduce:

> the widening gap between researchers and practitioners in mathematical education … If we are unsuccessful in this, research on mathematical education runs the risk of becoming barren dry swimming, while the practice of teaching runs the risk of becoming more naïve, narrow-minded, and inefficient than necessary and desirable. (from hand-out note)

After considering mathematics education research in the past century, Kilpatrick (2000) concluded that:

> the current practices in school mathematics that reformers are attempting to change have not themselves received experimental validation and that the available evidence from experience indicates that those practices are far from optimal. (p. 126)

Some mathematics educators argue that the design of a mathematics curriculum should be based on research. However, I believe that one should not decide whether a topic is or is not to be included in the curriculum because some studies find that some students could not grasp it after being taught by certain methods. This is so because all research findings, as well as educational theories, are not conclusive. In the future, more successful methods may be found to teach the so-called difficult topics to the target groups of students. Nevertheless, the dilemma of using or not using research is a controversial one and is highlighted by Hiebert (1999):

> The current debates about the future of mathematics education often lead to confusion about the role that research should play in settling disputes. On the one hand, researchers are called upon to resolve issues that really are about values and priorities, and on the other hand, research is ignored when empirical evidence is essential. (p. 3)

I wish to conclude this proposition with some comments by Kilpatrick (2000):

> in the next decade or so researchers in mathematics education … will recognise the value of developing a conception of research in which reliable evidence is gathered, justifications are offered, knowledge claims are tested, and the resulting knowledge is made public and can be independently verified. (ibid.)

Onward march FOMER!

Proposition 9: FOME is part and parcel of an overall futuristic education

It is clear from the cursory review above that FOME tends to focus more on the *M*athematics aspect and less on the *E*ducation aspect. Education is more than a collection of literacies. It should encompass the total development of a student:

- physical, e.g., use mathematical information on nutrition and exercise to promote health;
- cognitive, mathematics is an important but not the only component;
- emotional, mathematics attitude is much talked about but has not been emphasised in classroom teaching;
- moral, the values promoted by learning mathematics are rarely mentioned; see Proposition 4, and
- spiritual, has yet to be explored in FOME.

It has been said, "teachers should not teach the subject only, but also the whole child." At ICME-9, I attended the WGA3 (Mathematics Education in Senior Secondary School) sessions and listened to some very interesting ideas about rich mathematical experiences, problem solving and modelling, logic language, and thinking. These are important issues, but towards the end, I realised that most of the participants were not even thinking about the student as a whole person. At the last session of this workgroup, I raised the following concern. Students in senior secondary schools (15 to 19 years old) face many pressing issues: become more independent from their parents, handle relationships with the opposite sex, think about career and further studies, cope with influences from peers (e.g., drugs and alcohol) and the media (e.g., violence), understand who they are, and so on. Mathematics lessons take up about 15% of their school life and, in countries where mathematics tuition is common and heavy homework is given, learning mathematics may consume even more time and effort outside school hours. Mathematics educators need to extend their vision beyond mathematics per se and study how the rich mathematical learning experiences we offer in schools can help adolescents cope with these pressing issues to become "happy" and well-adjusted individuals. Indeed, these "heart" matters are far more important than academic results, and hopefully they will be addressed in FOME in the near future.

FOME must be integrated into the futuristic education system of each country, returning to a theme mentioned earlier under Proposition 2. This FOE is encapsulated as *Thinking Schools* in Singapore, *Smart Schools* in Malaysia, *Thoughtful Schools* in Brunei Darussalam, and other forms in different countries. Whatever labels are used, some common characteristics are emphasised:

- life-long learning, no one graduates, learning begins at pregnancy and completes only at the deathbed;
- on-time learning, learn mathematics whenever it is required;
- borderless learning, international in scope;

- resource-rich learning, not limited to only printed textbooks and workbooks; and
- active and creative citizenship.

Mathematics educators and teachers must talk to educators and teachers in science, language, social studies, arts, and so on, so that they *collectively* strive towards the same goals for FOE. Unfortunately, judging from the ways in which school curriculum is being designed all over the world and the conditions under which most teachers work, it is clear that this inter-disciplinary dialogue is sorely missing. Will mathematics educators take the lead in promoting such a dialogue with other educators and teachers?

Conclusion

Mathematics educators who engage in FOME need to become proactive and get involved by joining committees that make decisions about curriculum, developing materials and pedagogy that promote their visions, conducting workshops for not only teachers but also ministry officials, parents and the public to gather support for their visions, and so on. They can no longer be contented with arguing for their visions and let others work out the details, which may lead to a different outcome.

We are at the beginning of a new century (or millennium). We stand at a vintage point and can draw from the large corpus of research and practices in mathematics education and related fields, as we prepare to respond to the unprecedented challenges of educating our young people to understand and use mathematics in all aspects of their life. These young people will hopefully carry on our visions into the future:

> In the final scene of the movie, *Back to the Future,* Dr Brown's car floated in the air and he exclaimed: "In the future, there are no roads!" Fortunately, there are still many different roads to travel for FOME. The other papers in this special issue of *The Mathematics Educator* are the products of acknowledged experts in their fields. Read these papers carefully and be optimistic about the future!

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